Extensibility of Cayley Graphs of a Class of Clifford Semigroups

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Abstract: The semigroup extension is based on a known or simple semigroup class and extends to a class of semigroups by some method. The structure and properties of Cayley graphs of strong semilattices of left groups are studied, and a sufficient condition is given that a directed graph is a Cayley graph of strong semilattices of left groups. The $1\frac{1}{2}$- expandability and 2- expandability of *Cay (T, P)* graphs of Clifford semigroups with Γ being a circle are characterized. If the left group is restricted to be a group, the corresponding results of Cayley graphs of Clifford semigroups can be obtained, thus generalizing some main conclusions about Cayley graphs of such semigroups.

1. Introduction

The study of Cayley graphs of groups has always been a relatively active field in graph theory. At the same time, the study of Cayley graphs of semigroups has also caused great interest in graph theory scholars and semigroup theory scholars——Fuzzy sets [1], and introduces various algebraic operations and properties of fuzzy sets. Then the theoretical and applied research of fuzzy sets became a research hotspot in many fields such as mathematics, computer, economics and society and obtained rich research results [2-3]. There is a close connection between algebra and graph theory. At present, algebraic objects and graph theory are mainly connected through the self-homomorphic monoid of graphs, Cayley graphs, zero-factor graphs of rings or semigroups. The conditions required for the study of $1\frac{1}{2}$- expandability and 2-expandability of Cayley graphs are more complex than those for the study of expandability.

The Cayley graph of semigroups has been studied by many scholars, and some new and interesting results have been obtained in the past ten years. See Document [4]. Zhang discussed the Cayley graph of Clifford semigroups in Document [5]. In dealing with most practical problems, it is more difficult due to the uncertainty and fuzziness of information. After fuzzy sets and their extensions have been proposed one after another, a new direction is provided for the solution of such problems. However, these tools still have their limitations. Therefore, in order to find a better solution to these practical problems, it is necessary to continuously establish better tools. The study of Cayley graph and its generalization, an automorphism point transitive graph, has been very active for a long time [6]. Because Cayley graph generally has a long path, it has some very good combinatorial properties [7]. Cayley graphs are often used to construct some special graphs. In this paper, $1\frac{1}{2}$- extendibility and 2-extendibility of $Cay(T,P)$ graphs of Clifford semigroups with Γ being a circle are characterized.

2. Propaedeutics

This paper assumes that all sets are finite. First, some basic concepts and symbols are introduced. Graph refers to ordered triplet (Q,W,Φ), where non-empty set Q is called vertex set, W is called edge set, and W is a function of ordered pair or disordered pair cluster $Q\times Q$ of elements in Q. $\mathrm{Re}\,gS$ represents the set of regular elements of S, $Q(S)$ represents the set of idempotent elements

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of S, and $\langle Q(S)\rangle$ represents the subsemigroup generated by the set of idempotent elements. Let E be a semigroup. Set $C(S)=\{c\in S\mid sc,s\in S\}$ is called the center of S. The set of idempotent elements is denoted $E(s)$. $s\in S$ is called a regular element of S. If $y\in S$ makes $sys=s$. If every element in S is a regular element, then the semigroup is called a regular semigroup. Further, if $E(S)\subseteq C(S)$, then the regular semigroup is called a Clifford semigroup, that is, the idempotent element of S is interchangeable with all other elements.

The basic homomorphism theorem of groups is reviewed. Let H be a group and α be a full homomorphism mapping from H to group $H^{'}$. Homomorphism kernel $J=\{a\in H\mid b\alpha=z\alpha\}$(c is a constant element of H) is a normal subgroup of $H^{'}$ quotient groups H/J and $H^{'}$ of H pair J are isomorphic. H can be regarded as an extension of normal subgroup J, which is done by means of group H (or quotient group H/J) in some sense. We find that there is a similar situation for the homomorphic image U_X of inverse semigroup H and Munn representing α. First, we give the concept of kernel of homomorphic α.

3. Characterization of Complete Cayley Graphs on Completely Simple Semigroups

Theorem 3.1 Let $T=A[H,L,P,G]$ be a completely simple semigroup, P be a non-empty subset of T, and take two unequal elements (j,a,μ) and (q,e,γ) in T, then (j,a,μ), $(q,e,\gamma)\in D(Cay(T,P))$, if and only if $\mu=\gamma$ and $\theta\in P$ exist, making $(q,ed^{-1}p_{\theta j}^{-1},\theta)\in P$.

Proof: If $((j,a,\mu),(q,e,\gamma))\in D(Cay(T,P))$, then $(v,z,\theta)\in P$ exists so that $(v,z,\theta)(j,a,\mu)=(v,zp_{\theta i}d,\mu)=(q,e,\gamma)\Leftrightarrow\gamma=\mu,v=q$ and $zp_{\theta i}d=e,z=ed^{-1}p_{\theta i}^{-1}$.

Let $T=A[H,L,P,G]$ be a completely simple semigroup and P be a non-empty subset of T. According to theorem 3.1, if two elements in T are connected by edges in graph $Cay(T,P)$ then the two elements belong to the same ω class of T. For any $\gamma\in P$, the derived subgraph of vertex set T_μ in graph $Cay(T,P)$ is called N_μ, which is a branch of graph $Cay(T,P)$. obviously, for any two unequal $\mu,\gamma\in PX(N_\mu)\cap P(N_\gamma)$.

Theorem 3.2 Cayley graph $\Gamma=Cay(T,P)$ of Clifford semigroup $F=K\cup L$ is connected if and only if Cayley graph $\Gamma_2=Cay(L,K_2\cup\eta(P_1))$ of group L is connected.

Proof: If Cayley graph Γ_2 is connected and two vertices of Γ are taken, we need only prove that there is a path of Γ connecting the two vertices. According to the positions of the two vertices, we can divide into the following three situations:

(1) $\varphi_0,\varphi_1\in S(\Gamma_1)$

If $t\in T_2$ is taken, then $t\varphi_0=t\eta(\varphi_0),t\varphi_1=t\eta(\varphi_1)\in L$. since Γ_2 is connected, there is a path O connecting $t\varphi_0$ and $t\varphi_1$. thus $\varphi_0(t\varphi_0)O\varphi_1(t\varphi_1)$ is the path connecting φ_0 and φ_1.

$\varphi\in S(\Gamma_1),f\in S(\Gamma_2)$

If $t\in T_2$ is taken, there is $t\varphi=t\eta(\varphi)\in L$. Because Γ_2 is connected, there is a path O connecting $t\varphi$ and f. So $\varphi(t\varphi)Of$ is the path connecting φ and f.

(3) $f_0,f_1\in S(\Gamma_2)$

Since Γ_2 is connected, there is a way O of Γ_2 connecting f_0 and f_1.

Theorem 3.3 Let L be a commutative semigroup, and let $xDy \Leftrightarrow$ have $v,b>0$ and $x|y^b, b|x^b$, then D is the congruence in L, and $f: L \to L/D$ is the semilattice homomorphism of L. if $j: L \to L_1$ is also the semilattice homomorphism of L, then there is a unique homomorphism $e: L/D \to L_1$, making $j=ef$. conversely, if L has the maximum semilattice homomorphism G, then $G \cong L/D$.

Proof: D is the thinnest semilattice congruence of L. Since j is semilattice homomorphism, the primitive image classification of j is semilattice congruence Kerg, thus $D=Kerg$, thus there is a unique natural homomorphism $e: L/D \to L_1$, making $j=ef$. construct category W, object is binary group (F,m), where F is semilattice, $m: L \to F$ is semigroup homomorphism, morphism is homomorphism $f: F_1 \to F_2$, making $m_1 f = m_2$, then $(\overline{L}=L/D, f)$ is the starting object of W, if (G,i) is also the starting object, then $(G,i) \cong (L,f)$.

Theorem 3.4 Let S be a semilattice and the kernels I and S of Munn semigroups T_S on S are isomorphic.

Proof: Let $\beta \in T_S$, then β is the isomorphism of S_s to S_f. therefore, $\beta\beta^{-1}=\beta_s, \beta^{-1}\beta=\beta_f$, if $\beta \in I$, then any idempotent elemen T_S of β and ts can be exchanged. therefore, $\beta=\beta\beta^{-1}\beta=\beta_s\beta=\beta\beta_s$, so $\beta^{-1}\beta=\beta^{-1}\beta\beta_s$, i.e., $\beta_f=\beta f\beta_s$. and $\beta=\beta\beta^{-1}\beta=\beta\beta f=\beta f\beta$, so $\beta_s=\beta f\beta_s$, so $\beta_e=\beta_f$, thus $s=f$ obtained. Therefore, β is the automorphism of S_s. It is proved below that β is the constant isomorphism of S_s. If $b \in S_s, b\beta=l$ is set, $\beta_b\beta=\beta_l$. According to the multiplication rule of T_S, the domain of $\beta_b\beta$ should be $S(bs)$. Because $b \in S_s$, $bs=b$, $S(bs)=Sb$. Because β_l's domain is Sl, $b=l$. Therefore, β is the constant isomorphism of S_s. That is, $\beta=\beta_s \in T_S$.

4. Clifford Semigroup Packets of Separable Semigroups

Theorem 4.1 If Γ_2 is a circle and $D/V_{2\lambda}(U)^1 \neq \Phi$ then the Cayley graph Γ_2 of Clifford semigroups is not $1\frac{2}{2}-$ extendable or $2-$extendable.

Proof: Let's assume $\Gamma_2 = Cay(D, V_2 \cup \lambda(C_1)) = i_1 i_2 \cdots i_n i_0$ is $i_m \in D/V_{2\lambda}(U),\ m=0,1\cdots n,$ and $c(i_m)=1$. According to parity of $|U|+|D|$, it can be divided into the following two cases:

(1)If $|U|+|D|$ is odd, if $d=i_{m+1}i_{m+2}$ and vertex i_{m-1} are selected, then there is no perfect match of covering i_m with $\Gamma\text{-}i_{m-1}$.

(2) If $|U|+|D|$ is an even number and $d_1=i_{m-2}i_{m-1}, d_2=i_{m+1}i_{m+2},$ is selected, then there is no perfect match of Γ covering i_m.

Theorem 4.2 Let C_T be a separable $E-$fuzzy Clifford semigroup and $\forall_y \in C$ be idempotent, that is, the necessary and sufficient condition of $y^2=y$ is that y is the center of C_T.

Proof: The sufficiency of forensics. Let $\forall_y \in C$ be the center of C_T then $T(y^2,y) \geq t(y)$, and $T(y^2,y) \leq t(y)$ is known from the strictness of T, so $T(y^2,y)=t(y)=t(y,y)$, combined with separability, can obtain $y^2 = y$.

The necessity of re-proving. If $\forall_y \in C$ has $y^2 = y$, then $T(y, y^2) = t(y)$, so y is the center of C.

Inference 1 Let C_T be a separable $E-$fuzzy Clifford semigroup, $y, c(y) \in C$ and $c(y)$ are only related to y, if there is a weak Γ equivalence relation, $T(y, c(y)) \ge t(y)$ is sufficient and only $c(y) = y$ exists in the contemporary number C.

Proof: Adequacy. If $y, c(y) \in C$ and $c(y) = y$, there is $T(c(y), y) = t(y)$. Obviously, the weak Γ equivalence relation $T(y, c(y)) \ge t(y)$.

Necessity. If $\forall_y \in C$ has $T(c(y), y) \ge t(y)$ and $T(c(y), y) = t(y)$ is known in combination with the strictness of weak Γ equivalence relation, then $c(y) = y$.

Theorem 4.3 L is the J equivalent of N, L_γ is the J equivalent of T_E.

Proof: Let $L = L_1 \cap L_2$ (L_1 L_2 are each an F equivalent class and a K equivalent class of N). By Theorem 4.2, $L_1^{'} = L_{1\gamma}, L_2^{'} = L_{2\gamma}$ are each an F equivalent class and a K equivalent class of T_E. Thus $L^{'} = L_1^{'} \cap L_2^{'}$ is a J equivalent class of T_E. $L_\gamma = (L_1 \cap L_2)\gamma \subseteq L_{1\gamma} \cap L_{2\gamma} = L^{'}$. First assume that L has idempotent v, thus $v\gamma \in L^{'}$, so L and $L^{'}$ are both groups. Let $l^{'} \in L^{'}$, from $l_1^{'} \in L_1^{'}$, then $l_1 \in L_1$ makes $l_{1\gamma} = l^{'}$. Similarly $l_2 \in L_2$ makes $l_{2\gamma} = l^{'}$. Because $(l_1 l_1^{-1})\gamma = l^{'}(l^{'})^{-1} = v\gamma, (l_2 l_2^{-1})\gamma = l^{'}(l^{'})^{-1} = v\gamma$, so $(l_1 l_1^{-1})\gamma = (l_2 l_2^{-1})\gamma$. Because $l_1 l_1^{-1}$ and $l_2 l_2^{-1}$ are idempotent, so $l_1 l_1^{-1} = l_2 l_2^{-1}$. Similarly $l_1^{-1} l_1 = l_2^{-1} l_2$. Therefore l_1, l_2 are in the same J equivalence, because $l_1 \in L_1, l_2 \in L_2$, so l_1 and l_2 are all in, $l^{'} = l_{1\gamma} \in L_\gamma$, thus proving $L^{'} \subset L_\gamma$. So $L^{'} = L_\gamma$. Then suppose L is not a group, because $L \subseteq L_1$, let the J equivalent class X contained in L_1 be a group, then X_γ is the J equivalent class of T_E. According to green lemma, there is a, b in N to make $aX = L, bL = X$, so $(a\gamma)(X\gamma) = (L\gamma), (b\gamma)(L\gamma) = (X\gamma)$. So L_γ is also the J equivalent class of T_E.

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